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Influence of internal turbulent structure on intensity of velocity and temperature fluctuations of particles

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Abstract

The hypothesis of "independent averaging" suggested by Corrsin [J. Atmos. Sci. 20 (1963) 115] is generalized for investigating the influence of turbulent microstructure on the intensity of velocity and temperature fluctuations of inertial particles. It has been shown that parameters of turbulent motion and heat transfer in a dispersed phase also depend on dynamic and thermal relaxation times of particles. It is established that amplitudes of velocity and temperature fluctuations of the dispersed phase are expressed in terms of the Eulerian space-time correlation functions measured in the system of coordinates moving with mean velocity of fluid flow. Association between Lagrangian and Eulerian turbulent time macroscales of various types of flows was estimated. These parameters were evaluated on the basis of published experimental data. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Turbulent nonisothermal gas flows laden with particles or droplets occur in a wide range of technical applications, for example, in power engineering, chemical technological processes, and aviation design. They are also realized in some natural phenomena, such as pollutant transport and precipitation in the atmosphere.

The intensity of turbulent fluctuations in a dispersed phase is regulated by the dynamic inertia of particles and by their average velocity relative to the velocity of the carrier fluid phase. As the velocity drift of phases increases, both the energy of random motion of the particles, and the coefficient of turbulent diffusion of dispersed impurity decrease. This well-known effect of "crossing trajectories" is associated with a decrease in the contact time of particles with power-containing eddies of velocity fluctuations of the carrier fluid. It has been extensively investigated experimentally [1,2], by analytical methods [3–9] and by methods of direct numerical simulation (DNS) [10–12].

In the absence of steady velocity drift between the phases, as the dynamic relaxation time of particles increases, the energy of random motion of dispersed impurity decreases. At the same time, in accordance with a well-known statement of Chen (see, for instance, [13]) the coefficient of turbulent diffusion of particles (without allowance for the effect of "crossing trajectories") does not depend on particles inertia and is equal to the coefficient of turbulent diffusion of a passive (inertia less) substance. Also it was shown in papers by Reeks [6], Squires and Eaton [11], Pismen and Nir [14], that the stationary coefficient of turbulent diffusion of inertial particle is larger than the coefficient of turbulent diffusion of the passive substance. This behavior of the turbulent diffusivity of particles can be explained by the fact that the magnitudes of Lagrangian and Eulerian integral turbulent macroscales determined in a system of coordinates moving with the mean velocity of fluid flow are different [15].

The Lagrangian and Eulerian correlation functions differ fundamentally in the procedures for their determination. Both correlations are obtained by the method of averaging over an ensemble of realizations. The Lagrangian time correlation function the averaged product of instantaneous particle velocity components along their own trajectories. The Lagrangian time correlation functions reflect individual properties of the particles.

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Nomenclature

C_K, C_B	Kolmogorov and Batchelor constants
C_D	coefficient of hydraulic resistance of particles
c_p, c_g	heat capacities of the materials of particles and fluid phase, (J kg^{-1} K^{-1})
D^p_{ii}, D°_{ii}	turbulent diffusion coefficient of inertial particles and passive substance (m^2 s^{-1})
d_p	diameter of particles (m)
$E_{ij}(\boldsymbol{Y}, \xi)$	Eulerian correlation functions of velocity fluctuations of carrier phase (m^2 s^{-2})
$E_\theta(\boldsymbol{Y}, \xi)$	Eulerian correlation function of temperature fluctuations of carrier phase (K^2)
$\hat{E}_{ij}(\boldsymbol{k}, \xi)$	Fourier transformation of Eulerian velocity correlation functions (m^3 s^{-2})
$\hat{E}_\theta(\boldsymbol{k}, \xi)$	Fourier transformation of Eulerian temperature correlation functions (K^2 m)
f_{ii}, f_θ	response functions of particles to turbulent fluctuation of velocity and temperature of fluid phase
G_p	probability density function of particles transition
$\boldsymbol{k}$	vector in the space of wave numbers (m^{-1})
L_E, L_θ	integral Eulerian space macroscales of velocity and temperature fluctuation functions (m)
Nu	Nusselt number
Pr	Prandtl number of fluid phase
$\boldsymbol{R}_p$	radius-vector of a particle (m)
Re_p	particle Reynolds number
T_E, T_θ	Eulerian integral time macroscales of velocity and temperature fluctuations (m)
T^L_u, T^L_θ	Lagrangian integral time macroscales of velocity and temperature fluctuations (s)
T^p_{ii}, T^p_θ	integral time scales of correlation functions of velocity and temperature fluctuations of carrier phase along the trajectories of particle (s)
$\boldsymbol{U}$	actual velocity of the fluid phase (m s^{-1})
$\boldsymbol{U}_0$	average velocity of fluid flow (m s $^{-1}$)
$\boldsymbol{u}$	velocity of fluid phase fluctuations (m s^{-1})
$\boldsymbol{V}_p$	actual velocity of a particle (m s^{-1})
$\boldsymbol{v}_p$	fluctuation velocity of a particle (m s^{-1})
$\boldsymbol{W}$	sedimentation velocity of a particle (m s^{-1})
$\boldsymbol{X}_p$	random displacements of a particle (m)
α	ratio between sedimentation velocity of particles and r.m.s. fluid velocity, $\alpha = W/u$
β_E, β_θ	ratio between Lagrangian and Eulerian time macroscales of correlation functions of velocity and temperature fluctuations
$\Gamma_{ii}, \Gamma_\theta$	integral Lagrangian time scales of correlation functions of velocity and temperature fluctuations of particles velocity (s)
$\Delta(x)$	Heaviside stepwise function
δ_E, δ_θ	turbulent structural parameters characterizing velocity and temperature of fluid phase $\delta_u = uT_E/L_E$, $\delta_\theta = uT_\theta/L_\theta$
ε_u	turbulent dissipation rate of velocity fluctuation (m^2 s^{-3})
ε_θ	turbulent dissipation rate of temperature fluctuation (K^2 s^{-1})
ζ	ratio between the temperature and dynamic relaxation times of particles, $\zeta = \tau_\theta/\tau_u$
Θ_f, Θ_p	actual temperatures of carrier phase and particles (K)
Θ_0	average temperature of fluid phase (K)
θ_f, θ_p	temperature fluctuations of carrier phase and particles (K)
λ	Taylor space microscale (m)
ν	kinematic viscosity of fluid phase (m^2 s^{-1})
ρ_p, ρ_g	densities of particles and fluid phase material (kg m^{-3})
σ_{ii}	average squared velocity fluctuations of particles (m^2 s^{-2})
τ_K	Kolmogorov time microscale (s)
τ_u, τ_θ	times of dynamic and temperature relaxation of particles (s)
Φ_{ii}, Φ_θ	Lagrangian correlation functions for velocity and temperature fluctuations of particles
$\varphi(\boldsymbol{v})$	probability density function of particles velocity fluctuations (m s^{-1})
χ	ratio between the amplitude of velocity fluctuations and averaged flow velocity, $\chi = u/U_0$
Ψ_{ii}, Ψ_θ	Eulerian correlation functions for velocity and temperature fluctuations of fluid phase
$\Psi^p_{ii}, \Psi^p_\theta$	correlation functions for velocity and temperature fluctuations of carrier phase along a particle trajectory
Ψ°_{ii}	Lagrangian correlation function of velocity fluctuation of passive substance
Ω_E	parameter of particle inertia, $\Omega_E = \tau_u/T_E$
ω_E, ω_θ	frequencies of velocity and temperature fluctuations in the Eulerian variables (s^{-1})

The Eulerian correlation functions are determined at a fixed point of space. They can be measured in the system of coordinates connected with the mean fluid velocity of the flow, or in a fixed "laboratory" system of coordinates. In the calculation of the integral time scale of Eulerian correlation functions, the products of the random velocity components of the fluid at a fixed point of space are averaged. Space-time Eulerian correlation functions give the presentation about the collective random behavior of the ensemble of fluid phase microparticles passing trough the fixed point of space.

The coefficient of turbulent diffusion of a passive substance is determined by the Lagrangian correlation functions. In the case of inertial particles, however, the parameters of turbulence of the carrier phase along a particle trajectory are combined with both Lagrangian and Eulerian correlation functions.

Intensive theoretical studies were carried out by analytical methods [16–21] and by DNS methods [22,23] to determine a relation between Lagrangian and Eulerian correlation functions. There are also some experimental papers, for instance, Sato and Yamamoto [24] and experimental data collected in the paper of Middleton [21] on the subject of study. It was found that the Lagrangian time integral macroscale is less than the Eulerian time integral macroscale in the system of coordinates joined with the average velocity of the fluid flow.

This result can be physically interpreted as follows. The random velocity field in the fixed point of space (the Eulerian variables) is an agglomeration of the collective correlated non-local interaction in the surrounding fluid. The effect of non-local interaction in the turbulent velocity field is the result of pressure fluctuations. If the labeled particle (in Lagrangian variables) moves in the fluid, the correlation of velocity fluctuations of the individual particle (along its trajectory) in this random field breaks down [20].

The theoretical investigations of the relationship between Lagrangian and Eulerian macroscales and turbulent diffusivity of passive substance [18,25–27] are based on the hypothesis "of independent averaging" suggested by Corrsin [28]. Within the framework of this hypothesis it is proposed to perform averaging of the probability density function of the particle transition independently from the Eulerian two-point (space–time) velocity correlation functions. The arguments for the hypothesis of independent averaging are based on some modern concepts of the turbulence microstructure and the data of DNS and experiments. It is worthwhile to note, that the above investigation results are limited to the case of homogeneous isotropic turbulence.

It was found in experimental papers by Sato and Yamamoto [24] and Krasheninnikov and Secundov [29] that the relation between Lagrangian and Eulerian space and time macroscales is determined by the type of turbulent flow in question. In this connection, it is necessary to expect direct relationship between parameters of particle random motion and microstructure of turbulence of the flow.

The intensity of temperature fluctuations of particles is determined by both particle dynamic and thermal relaxation times. The latter is connected with thermo physical properties of material of dispersed and fluid phases. The DNS method used by Jaberi [30] illustrates that the temperature fluctuations of particles depend on their inertia and microstructure of turbulent temperature fluctuations of the carrier fluid.

Modern theoretical papers devoted to the determination of the intensity of turbulent velocity and temperature fluctuations of particles typically used a simplified approach. This approximation is based on employing only one time macroscale of turbulence (Lagrangian or Eulerian) (see for instance, [31]). Since these scales are essentially different in magnitude, the calculations performed by using such an approach may result in significant errors.

In the present paper, the Corrsin's hypothesis "of independent averaging" is generalized to velocity and temperature fluctuations of inertial particles. Closure expressions are presented for the inertial particles Lagrangian correlation functions through the Eulerian velocity and temperature correlation functions of the carrier fluid. Some integral parameters depending on the flow microstructure are determined from experimental data obtained for various types of flows. It has been found that the intensity of random motion and heat transfer in the dispersed phase is related to the dynamic and thermal inertia of particles and to the structure of the flow.

2. Particle velocity and temperature fluctuations

We consider particles of spherical shape, and with size smaller than the Kolmogorov space turbulence microscale. This situation may be realized in a gas flow with particles or droplets. In the equation of particles motion we take into account only viscous drag and gravity

$$\frac{\mathrm{d}V_{pi}(t)}{\mathrm{d}t} = \frac{1}{\tau_u}[U_i(\boldsymbol{R}_\mathrm{p}(t),t) - V_{pi}(t) + W_i], \quad \frac{\mathrm{d}R_{pi}}{\mathrm{d}t} = V_{pi}. \quad (1)$$

The coefficient of heat conductivity of the particle material exceeds the coefficient of heat conductivity of the carrier fluid. Therefore, temperature distribution over the volume of the particle is assumed to be uniform. In this situation, the equation of heat transfer for the particles has the following form:

$$\frac{\mathrm{d}\Theta_\mathrm{p}(t)}{\mathrm{d}t} = \frac{1}{\tau_\theta}[\Theta_\mathrm{f}(\boldsymbol{R}_\mathrm{p}(t),t) - \Theta_\mathrm{p}(t)]. \quad (2)$$

The dynamic and thermal relaxation times of the particles τ_u and τ_θ depend on the relative velocity between the particles and fluid phase.

In the following we analyze a statistically stationary and homogeneous turbulent flow of the carrier phase. It is assumed that the residence time of particles in the flow considerably exceeds the integral time scale of turbulence, and both the dynamic and thermal relaxation times of particles $t \gg (T_{\rm E}, \tau_u, \tau_\theta)$. After the procedure of averaging over an ensemble of turbulent realizations we separate average and fluctuating components of velocity and temperature of fluid and particles

$$\boldsymbol{U}(\boldsymbol{x},t) = \boldsymbol{U}_0 + \boldsymbol{u}(\boldsymbol{x},t), \quad \Theta_{\rm f}(\boldsymbol{x},t) = \Theta_0 + \theta_{\rm f}(\boldsymbol{x},t),$$

$$\boldsymbol{V}_{\rm p}(t) = \boldsymbol{U}_0 + \boldsymbol{W} + \boldsymbol{v}_{\rm p}(t), \quad \Theta_{\rm p}(t) = \Theta_0 + \theta_{\rm p}(t).$$

The statistical characteristics of the dispersed phase are described by correlation functions of velocity $\Phi_{ii}(s)$ and temperature $\Phi_\theta(s)$ fluctuations of particles along their own trajectories. We name these correlations also as Lagrangian correlation functions of particles

$$\langle v_{pi}(t)v_{pi}(t+s)\rangle = \langle v_{\rm p}^2\rangle\Phi_{ii}(s), \tag{3}$$

$$\langle \theta_{\rm p}(t)\theta_{\rm p}(t+s)\rangle = \langle \theta_{\rm p}^2\rangle\Phi_\theta(s). \tag{4}$$

Here and below repeated indexes do not mean summation.

The following expressions for Lagrangian correlation functions of the particles follow from the equations of motion and heat transfer (1) and (2):

$$\begin{aligned}\langle v_{pi}(t_1)v_{pi}(t_2)\rangle &= \frac{1}{\tau_u^2}\int_0^{t_1} {\rm d}s_1 \exp\left(-\frac{t_1-s_1}{\tau_u}\right) \\ &\quad\times \int_0^{t_2} {\rm d}s_2 \exp\left(-\frac{t_2-s_2}{\tau_u}\right) \\ &\quad\times \langle u_i(\boldsymbol{R}_{\rm p}(s_1),s_1)u_i(\boldsymbol{R}_{\rm p}(s_2),s_2)\rangle \\ &= \frac{\langle u_i^2\rangle}{\tau_u^2}\int_0^{t_1} {\rm d}s_1 \exp\left(-\frac{t_1-s_1}{\tau_u}\right) \\ &\quad\times \int_0^{t_2} {\rm d}s_2 \exp\left(-\frac{t_2-s_2}{\tau_u}\right)\Psi_{ii}^{\rm p}(s_1,s_2),\end{aligned} \tag{5}$$

$$\begin{aligned}\langle \theta_{\rm p}(t_1)\theta_{\rm p}(t_2)\rangle &= \frac{1}{\tau_\theta^2}\int_0^{t_1} {\rm d}s_1 \exp\left(-\frac{t_1-s_1}{\tau_\theta}\right)\int_0^{t_2} {\rm d}s_2 \\ &\quad\times \exp\left(-\frac{t_2-s_2}{\tau_\theta}\right)\langle\theta_{\rm f}(\boldsymbol{R}_{\rm p}(s_1),s_1)\theta_{\rm f} \\ &\quad\times(\boldsymbol{R}_{\rm p}(s_2),s_2)\rangle \\ &= \frac{\langle\theta_{\rm f}^2\rangle}{\tau_\theta^2}\int_0^{t_1} {\rm d}s_1 \exp\left(-\frac{t_1-s_1}{\tau_\theta}\right) \\ &\quad\times \int_0^{t_2} {\rm d}s_2 \exp\left(-\frac{t_2-s_2}{\tau_\theta}\right)\Psi_\theta^{\rm p}(s_1,s_2).\end{aligned} \tag{6}$$

Here velocity and temperature correlation functions of carrier phase along the particle trajectory are expressed in terms of the Eulerian correlation functions

$$\begin{aligned}\langle u_i^2\rangle\Psi_{ii}^{\rm p}(s_1,s_2) &= \int {\rm d}\boldsymbol{x}_1\int {\rm d}\boldsymbol{x}_2\langle u_i(\boldsymbol{x}_1,s_1)u_i(\boldsymbol{x}_2,s_2) \\ &\quad\times\delta(\boldsymbol{x}_1-\boldsymbol{R}_{\rm p}(s_1))\delta(\boldsymbol{x}_2-\boldsymbol{R}_{\rm p}(s_2))\rangle,\end{aligned} \tag{7}$$

$$\begin{aligned}\langle \theta_{\rm f}^2\rangle\Psi_\theta^{\rm p}(s_1,s_2) &= \int {\rm d}\boldsymbol{x}_1\int {\rm d}\boldsymbol{x}_2\langle \theta_{\rm f}(\boldsymbol{x}_1,s_1)\theta_{\rm f}(\boldsymbol{x}_2,s_2) \\ &\quad\times\delta(\boldsymbol{x}_1-\boldsymbol{R}_{\rm p}(s_1))\delta(\boldsymbol{x}_2-\boldsymbol{R}_{\rm p}(s_2))\rangle.\end{aligned} \tag{8}$$

In the case of homogeneous turbulent flow, correlations (7) and (8) are approximated by statistically stationary random processes

$$\Psi_{ii}^{\rm p}(s_1,s_2) = \Psi_{ii}^{\rm p}(\xi),\ \ \Psi_\theta^{\rm p}(s_1,s_2) = \Psi_\theta^{\rm p}(\xi) \quad (\xi = |s_1-s_2|). \tag{9}$$

As a result of some calculations, taking into account the Eqs. (5)–(8) with regard to statistical stationarity assumption (9), we receive expressions for functions of velocity and temperature fluctuations of particles along their own trajectories (Lagrangian correlations for particles)

$$\begin{aligned}\langle v_{pi}(t)v_{pi}(t+s)\rangle &= \langle v_{pi}^2\rangle\Phi_{ii}(s) \\ &= \frac{\langle u_i^2\rangle}{2\tau_u}\int_0^\infty\left[\exp\left(-\frac{|s+\xi|}{\tau_u}\right)\right. \\ &\quad\left.+\exp\left(-\frac{|s-\xi|}{\tau_u}\right)\right]\Psi_{ii}^{\rm p}(\xi)\,{\rm d}\xi,\end{aligned} \tag{10}$$

$$\begin{aligned}\langle \theta_{\rm p}(t)\theta_{\rm p}(t+s)\rangle &= \langle \theta_{\rm p}^2\rangle\Phi_\theta(s) \\ &= \frac{\langle\theta_{\rm f}^2\rangle}{2\tau_\theta}\int_0^\infty\left[\exp\left(-\frac{|s+\xi|}{\tau_\theta}\right)\right. \\ &\quad\left.+\exp\left(-\frac{|s-\xi|}{\tau_\theta}\right)\right]\Psi_\theta^{\rm p}(\xi)\,{\rm d}\xi.\end{aligned} \tag{11}$$

Note that formulas (10) and (11) are valid for large residence times of the dispersed phase in the flow $t \gg (T_{\rm E}, \tau_u, \tau_\theta)$, when fluctuations of the parameters of the fluid phase at the particles trajectories may be represented as a statistically stationary random process.

It is seen from Eqs. (10) and (11), that Lagrangian correlation functions of particles decrease exponentially as the relative time increases $s \to \infty$: $\Phi_{ii}(s) \approx \exp(-s/\tau_u)$ and $\Phi_\theta(s) \approx \exp(-s/\tau_\theta)$. As $s = 0$ expressions (10) and (11) result in formulas for the intensities of velocity and temperature fluctuations of the dispersed phase

$$\langle v_i^2\rangle = \sigma_{ii} = \frac{\langle u_i^2\rangle}{\tau_u}\int_0^\infty \exp\left(-\frac{\xi}{\tau_u}\right)\Psi_{ii}^{\rm p}(\xi)\,{\rm d}\xi, \tag{12}$$

$$\langle \theta_{\rm p}^2\rangle = \frac{\langle\theta_{\rm f}^2\rangle}{\tau_\theta}\int_0^\infty \exp\left(-\frac{\xi}{\tau_\theta}\right)\Psi_\theta^{\rm p}(\xi)\,{\rm d}\xi. \tag{13}$$

It is seen from Eqs. (12) and (13) that for inertial particles $\tau_u, \tau_\theta \gg T_E$ amplitudes of velocity and temperature fluctuations of particles are proportional to corresponding ratios between relaxation times of particles and the integral temporal macroscales of velocity and temperature fluctuations of carrier phase along the particle trajectories

$$T_{ii}^{p} = \int_0^\infty \Psi_{ii}^{p}(\xi)\mathrm{d}\xi, \quad T_\theta^{p} = \int_0^\infty \Psi_\theta^{p}(\xi)\,\mathrm{d}\xi, \tag{14}$$

$$\langle v_i^2 \rangle \propto \langle u_i^2 \rangle T_{ii}^{p}/\tau_u, \quad \langle \theta_p^2 \rangle = \langle \theta_f^2 \rangle T_\theta^{P}/\tau_\theta. \tag{15}$$

3. Time scales of fluid velocity and temperature correlation functions

The following expressions for relations between Lagrangian integral time scales of particles and the time scales of functions of velocity and temperature fluctuations of the carrier phase along the particle trajectory may be obtained after integration of Eqs. (10) and (11):

$$\Gamma_{ii} = \int_0^\infty \Phi_{ii}(s)\,\mathrm{d}s = \langle u_i^2 \rangle \langle v_{pi}^2 \rangle^{-1} T_{ii}^{p}, \tag{16}$$

$$\Gamma_\theta = \int_0^\infty \Phi_\theta(s)\,\mathrm{d}s = \langle \theta_f^2 \rangle \langle \theta_p^2 \rangle^{-1} T_\theta^{p}. \tag{17}$$

For inertial particles with $\tau_u, \tau_\theta \gg T_E$, as determined from (15), integral time scales of Lagrangian correlation functions of velocity and temperature fluctuations of particles tend to the corresponding relaxation times of the particles $\Gamma_{ii} \propto \tau_u$, $\Gamma_\theta \propto \tau_\theta$.

Consider in greater detail the correlations of the function of fluid phase velocity fluctuations along the particle trajectory (7). The behavior of temperature correlation function of fluid phase at the particles trajectories is similar.

An essential way to study dispersed phase response functions of turbulence fluctuations of the carrier fluid is based on the analysis of statistical flow parameters in the coordinate frame moving with the carrier phase averaged velocity. For a statistically stationary homogeneous turbulence, expression (7) can be written in the following form:

$$\begin{aligned}\langle u_i^2 \rangle \Psi_{ii}^{p}(\xi) &= \int \langle u_i(\boldsymbol{x},t) u_i(\boldsymbol{x}+\boldsymbol{Y}, t+\xi)\delta(\boldsymbol{Y} - \boldsymbol{R}_p(\xi))\rangle\,\mathrm{d}\boldsymbol{Y} \\ &= \int \langle u_i(\boldsymbol{x},t) u_i(\boldsymbol{x}+\boldsymbol{Y}, t+\xi) G_p(\boldsymbol{Y},\xi)\rangle\,\mathrm{d}\boldsymbol{Y},\end{aligned}$$

where function $G_p(\boldsymbol{Y},\xi) = \delta(\boldsymbol{Y} - \boldsymbol{R}_p(\xi))$ is an instantaneous probability density function of particle transition on the distance $\boldsymbol{Y}$ during time interval ξ (in the moving system of coordinates).

Due to particle inertia, this function is connected with the more general probability density transition function $G_p(\boldsymbol{Y},\xi|\boldsymbol{v}_0)$ that depends on the particle initial velocity $\boldsymbol{v}_0$ at the initial time $\xi = 0$

$$\begin{aligned}G_p(\boldsymbol{Y},\xi|\boldsymbol{v}_0) = \delta\Bigg\{ \boldsymbol{Y} - \underbrace{\tau_u \boldsymbol{v}_0\left[1 - \exp\left(-\frac{\xi}{\tau_u}\right)\right]}_{\text{I}} - \underbrace{\xi \boldsymbol{W}}_{\text{II}} \\ - \underbrace{\int_0^\xi \mathrm{d}s\left[1 - \exp\left(-\frac{\xi - s}{\tau_u}\right)\right]\boldsymbol{u}(\boldsymbol{R}_p(s),s)}_{\text{III}}\Bigg\}.\end{aligned} \tag{18}$$

Expression (18) takes into account particle inertial motion with initial velocity of particle (I), displacement due to the particle average relative velocity (II) (in the moving system of coordinates), and turbulent transition with velocity fluctuation of carrier phase (III). The detailed derivation of formula (18) is provided in Appendix A.

The random motion of particles with dynamic relaxation time $\tau_u \propto T_E$ is determined by integral action of the macroscale structure of turbulence and depends slightly on small-scale (high-frequency) velocity fluctuations of the fluid phase. In this case, one can generalize the hypothesis of "independent averaging" by Corrsin [28] for flow and heat transfer of inertial particles. Following the assumption of independent averaging between the Eulerian correlation functions of velocity and temperature fluctuations of fluid phase and probability density transition function of particles, one can write following expressions:

$$\langle u_i^2 \rangle \Psi_{ii}^{p}(\xi) = \int \mathrm{d}\boldsymbol{v}_0 \int \mathrm{d}\boldsymbol{Y} \varphi(\boldsymbol{v}_0) E_{ii}(\boldsymbol{Y},\xi)\langle G_p(\boldsymbol{Y},\xi|\boldsymbol{v}_0)\rangle, \tag{19}$$

$$\langle \theta_f^2 \rangle \Psi_\theta^{p}(\xi) = \int \mathrm{d}\boldsymbol{v}_0 \int \mathrm{d}\boldsymbol{Y} \varphi(\boldsymbol{v}_0) E_\theta(\boldsymbol{Y},\xi)\langle G_p(\boldsymbol{Y},\xi|\boldsymbol{v}_0)\rangle, \tag{20}$$

where Eulerian correlation functions of velocity and temperature fluctuations of the fluid phase in (19) and (20) are determined in the system of coordinates driven together with the average velocity of the carrier phase

$$E_{ij}(\boldsymbol{Y},\xi) = \langle u_i(\boldsymbol{x}_1,s_1) u_j(\boldsymbol{x}_2,s_2)\rangle,$$

$$E_\theta(\boldsymbol{Y},\xi) = \langle \theta_f(\boldsymbol{x}_1,s_1)\theta_f(\boldsymbol{x}_2,s_2)\rangle,$$

$$\boldsymbol{Y} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \xi = |s_1 - s_2|,$$

where the probability density function of the distribution of velocity fluctuations of particles has the Gaussian form

$$\varphi(\boldsymbol{v}) = \prod_{i=1}^{3}(2\pi\sigma_{ii})^{-1/2}\exp\left(-\frac{v_i^2}{2\sigma_{ii}}\right), \quad \sigma_{ii} = \langle v_{pi}^2\rangle. \tag{21}$$

Note that Corrsin's hypothesis [28] is widely used for the analysis of turbulent diffusion of passive substances (for example, [16,17,19–21,25,27]). Therefore, the equations presented below for less inertial particles with $\tau_u < T_{\mathrm{E}}$ are a reasonable estimation for statistical parameters of the dispersed phase.

It is convenient to investigate the effect of the turbulent microstructure on the random velocity and temperature fluctuations of the dispersed phase by using the following spectral representation:

$$E_{ij}(\boldsymbol{Y}, \xi) = \int \hat{E}_{ij}(\boldsymbol{k}, \xi)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{Y})\mathrm{d}\boldsymbol{k}, \tag{22}$$

$$E_{\theta}(\boldsymbol{Y}, \xi) = \int \hat{E}_{\theta}(\boldsymbol{k}, \xi)\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{Y})\,\mathrm{d}\boldsymbol{k}. \tag{23}$$

Using Eqs. (18)–(23), we obtain the following expressions for correlation functions of velocity and temperature fluctuations of the carrier phase along a particle trajectory:

$$\langle u_i^2\rangle \Psi_{ii}^{\mathrm{p}}(\xi) = \int \hat{E}_{ii}(\boldsymbol{k}, \xi)\exp\left(-\mathrm{i}k_n W_n\xi - \frac{1}{2}k_n^2\langle\Lambda_n^2(\xi)\rangle\right)\mathrm{d}\boldsymbol{k}, \tag{24}$$

$$\langle \theta_{\mathrm{f}}^2\rangle \Psi_{\theta}^{\mathrm{p}}(\xi) = \int \hat{E}_{\theta}(\boldsymbol{k}, \xi)\exp\left(-\mathrm{i}k_n W_n\xi - \frac{1}{2}k_n^2\langle\Lambda_n^2(\xi)\rangle\right)\mathrm{d}\boldsymbol{k}, \tag{25}$$

where a summation on index n in the exponential terms is applied.

$$\langle\Lambda_n^2(\xi)\rangle = \underbrace{\sigma_{nn}\tau_u^2\left[1 - \exp\left(-\frac{\xi}{\tau_u}\right)\right]^2}_{\mathrm{I}} + \underbrace{\langle Y_n^2(\xi)\rangle}_{\mathrm{II}} \tag{26}$$

$$\begin{aligned}\langle Y_n^2(\xi)\rangle = \langle u_n^2\rangle\Bigg\{&2\int_0^{\xi}\mathrm{d}s(\xi - s)\Psi_{nn}^{\mathrm{p}}(s) \\ &- \tau_u\int_0^{\xi}\mathrm{d}s\left[1 - \exp\left(-\frac{\xi - s}{\tau_u}\right)\right] \\ &\times\left[2 - \exp\left(-\frac{s}{\tau_u}\right) + \exp\left(-\frac{\xi}{\tau_u}\right)\right]\Psi_{nn}^{\mathrm{p}}(s)\Bigg\},\end{aligned} \tag{27}$$

where term $\langle\Lambda_n^2(\xi)\rangle$ is a squared particle displacement due to inertial travel (I) and turbulent transfer with energy containing eddies (II).

The detailed derivation of expressions (24)–(27) is given in Appendix B. The exponential factor in (24) and (25) takes into account effects of the average velocity drift between phases and the contribution of turbulent fluid internal microstructure on the random motion and heat transfer in the dispersed phase. From Eq. (14) it is clear, that the characteristic scale of variation of the variable ξ for correlation function in (27) is of the order of the integral time scale T_{nn}^{p}.

It follows from (27) that the squared displacement of inertialess particles ($\tau_u \to 0$) without any average velocity drift ($\boldsymbol{W} = 0$) is equal to the squared displacement of a passive substance:

$$\langle Y_n^2(\xi)\rangle = 2\langle u_n^2\rangle\int_0^{\xi}(\xi - s)\Psi_{nn}^{\circ}(s)\,\mathrm{d}s.$$

Here the function $\Psi_{nn}^{\circ}(s)$ is equal to the Lagrangian correlation function of velocity fluctuations of a passive substance, at $\tau_u \to 0$. The Eqs. (24) and (25) approximately describe a relation between the Lagrangian correlation function of velocity and temperature fluctuations of microparticles of the fluid medium and the corresponding space-time Eulerian correlation functions.

For inertial particles $\tau_u \gg T_{nn}^{\mathrm{p}}$ the squared displacement of particles decreases $\langle\Lambda_n^2\rangle \approx \langle u_n^2\rangle(T_{nn}^{\mathrm{p}})^3/\tau_u$. In this case, if there is no average velocity drift of phases, as is obvious from Eqs. (24)–(27), the correlation functions of velocity and temperature fluctuations along the trajectory of particle are close to the corresponding space–time Eulerian correlation functions.

Formulas (7), (8), (10), (11) and (24)–(27) describe, in a closed form, turbulence of particles taking into account their dynamic and thermal inertia, relative velocity between phases, and the microstructure of the flow. Expression for velocity and temperature correlation functions of the carrier phase along the particle trajectory in (24) and (25) can be obtained in the closed form by approximation of the integrals in (27). For this purpose we used in (27) a simple approximation for velocity correlation function of carrier phase along the particle trajectory

$$\Psi_{nn}^{\mathrm{p}}(s) = \Delta(T_{nn}^{\mathrm{p}} - s). \tag{28}$$

After substitution (28) in expression (27) we find for $\langle\Lambda_n^2(T_{nn}^{\mathrm{p}})\rangle$ the following expression:

$$\begin{aligned}\langle\Lambda_n^2\rangle = \sigma_{nn}\tau_u^2&\left[1 - \exp\left(-\frac{T_{nn}^{\mathrm{p}}}{\tau_u}\right)\right]^2 + \langle u_n^2\rangle\Bigg\{(T_{nn}^{\mathrm{p}})^2 \\ &- \tau_u^2\Bigg[\frac{2T_{nn}^{\mathrm{p}}}{\tau_u}\left(1 + \exp\left(-\frac{T_{nn}^{\mathrm{p}}}{\tau_u}\right)\right) \\ &- \left(3 + \exp\left(-\frac{T_{nn}^{\mathrm{p}}}{\tau_u}\right)\right)\left(1 - \exp\left(-\frac{T_{nn}^{\mathrm{p}}}{\tau_u}\right)\right)\Bigg]\Bigg\}.\end{aligned} \tag{29}$$

Note that the time scale of velocity fluctuations of the fluid phase on the particle trajectory T_{nn}^{p} depends in a self-consistent way on the Eulerian correlation functions in (24) and (25) and takes into account particle inertia and velocity drift between phases.

The response functions of particles to temperature fluctuations on the carrier phase is determined not only by the time of their dynamic relaxation, but also by the ratio between heat capacity values of the material of particles and fluid phase, as well by the Prandtl number of the carrier phase (see Appendix C). An increase in thermal inertia of particles leads to a weaker dependence of temperature fluctuations of particles on the microstructure of turbulence. To take into account this effect, the scale of variation of the variable ξ in (26) for the thermal fluctuation (25) is approximated by the quantity $T_\theta^{\mathrm{p}}(\Gamma_{nn}/\Gamma_\theta)$. An increase in the time of thermal relaxation of particles causes an increase in the Lagrangian timescale of temperature fluctuations of particles Γ_θ and, as a result, a decrease in the contribution of small-scale fluctuations of carrier phase. In this case, if expression (29) is used for calculating the temperature correlation functions in (26), the parameter T_{nn}^{p} is replaced by $T_\theta^{\mathrm{p}}(\Gamma_{nn}/\Gamma_\theta)$.

4. Approximation of Eulerian correlation functions

For further analysis we approximate the space-time Eulerian correlation functions of velocity and temperature fluctuations of the fluid (in the system of coordinates moving with the averaged fluid velocity) in the following form:

$$\hat{E}_{ij}(\boldsymbol{k},s) = \langle u_i u_j\rangle \frac{16}{(2\pi)^{3/2}} \frac{k^2}{k_{\mathrm{E}}^5}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \times \exp\left[-2\left(\frac{k}{k_{\mathrm{E}}}\right)^2 - \frac{(\omega_{\mathrm{E}} s)^2}{2}\right], \quad (30)$$

$$\hat{E}_\theta(\boldsymbol{k},s) = \left(\frac{2}{\pi}\right)^{3/2} \frac{\langle\theta_{\mathrm{f}}^2\rangle}{k_\theta^3} \exp\left[-2\left(\frac{k}{k_\theta}\right)^2 - \frac{(\omega_\theta s)^2}{2}\right], \quad k^2 = k_i k_i. \quad (31)$$

The functions (30) and (31) are satisfied to the following normalization conditions:

$$\int \hat{E}_{ij}(\boldsymbol{k},0)\,\mathrm{d}\boldsymbol{k} = \langle u_i u_j\rangle, \quad \int \hat{E}_\theta(\boldsymbol{k},0)\,\mathrm{d}\boldsymbol{k} = \langle\theta_{\mathrm{f}}^2\rangle.$$

The integral space and time scales of the distributions (30) and (31) are determined as

$$L_{\mathrm{E}} = (2\pi)^{1/2}/k_{\mathrm{E}}, \quad T_{\mathrm{E}} = (\pi/2)^{1/2}/\omega_{\mathrm{E}},$$
$$L_\theta = (2\pi)^{1/2}/k_\theta, \quad T_{\mathrm{E}} = (\pi/2)^{1/2}/\omega_\theta.$$

The choice of functions in the form of (30) and (31) makes it possible to obtain analytical formulas. This simplifies further analysis considerably.

It should be noted that the spectral functions (30) and (31) do not reflect distribution of intensity of velocity and temperature fluctuations in the inertial and inertial-convective regions of the turbulent spectra. However, the distributions (30) and (31) approximate well the behavior of velocity and temperature fluctuations in the power-containing part of the spectra (see, for instance, [32,33]).

Substituting (30) and (31) into Eqs. (24) and (25), we find expressions for the correlation functions of velocity and temperature fluctuations of the fluid phase along the trajectory of a particle taking into account the relative velocity between phases ($W_j = W\delta_{1j}$)

$$\Psi_{11}^{\mathrm{p}}(\xi) = \eta_1^{-1}\eta_2^{-4} \exp\left[-\frac{\omega_{\mathrm{E}}^2\xi^2}{2}\left(1 + \frac{W^2 k_{\mathrm{E}}^2}{4\omega_{\mathrm{E}}^2\eta_1^2}\right)\right], \quad (32)$$

$$\Psi_{22}^{\mathrm{p}}(\xi) = \frac{\Psi_{11}^{\mathrm{p}}(\xi)}{2}\left[1 + \frac{\eta_2^2}{\eta_1^2}\left(1 - \frac{W^2\xi^2 k_{\mathrm{E}}^2}{4\eta_1^2}\right)\right], \quad (33)$$

$$\Psi_\theta^{\mathrm{p}}(\xi) = \mu_1^{-1}\mu_2^{-2} \exp\left[-\frac{\omega_\theta^2\xi^2}{2}\left(1 + \frac{W^2 k_\theta^2}{4\omega_\theta^2\mu_1^2}\right)\right], \quad (34)$$

$$\eta_i^2 = 1 + \pi\langle\Lambda_i^2\rangle/(2L_{\mathrm{E}}^2), \quad \mu_i^2 = 1 + \pi\langle\Lambda_i^2\rangle/(2L_\theta^2), \quad (35)$$

where coordinate axis $i = 1$ is directed along, and axis $i = 2$ transverse to; the average velocity drift between phases.

For velocity correlation functions, formulas (32) and (33) are similar to those found earlier in [9]. It is seen from expressions (32)–(34) that increase in the drift velocity of phases causes an intensive decay of the correlation functions of velocity and temperature fluctuations of carrier phase along the particle trajectory (the effect of "crossing trajectories").

Using formulas (16), (17) and (32)–(34) we determine the time macroscales of functions of velocity and temperature fluctuations of fluid phase along the particle trajectory

$$T_{11}^{\mathrm{p}} = \frac{T_{\mathrm{E}}}{\eta_1\eta_2^4}\left[1 + \left(\frac{W\,T_{\mathrm{E}}}{L_{\mathrm{E}}\eta_1}\right)^2\right]^{-1/2}, \quad (36)$$

$$T_{22}^{\mathrm{p}} = \frac{T_{11}^{\mathrm{p}}}{2}\left\{1 + \frac{\eta_2^2}{\eta_1^2}\left[1 + \left(\frac{W\,T_{\mathrm{E}}}{L_{\mathrm{E}}\eta_1}\right)^2\right]^{-1}\right\}, \quad (37)$$

$$T_\theta^{\mathrm{p}} = \frac{T_\theta}{\mu_1\mu_2^2}\left[1 + \left(\frac{W\,T_\theta}{L_\theta\mu_1}\right)^2\right]^{-1/2}. \quad (38)$$

In the calculation of the intensity of fluctuations of dispersed phase parameters (12) and (13), the corresponding correlation function of the fluid velocity fluctuations along the particle trajectory is approximated by formula (28). The analogous form approximates the correlation function of temperature fluctuations of the fluid along the particle trajectory $\Psi_\theta^{\rm p}(s) = \Delta(T_\theta^{\rm p} - s)$. As a result, we find response functions of particles on velocity and temperature fluctuations of carrier phase

$$\langle \sigma_{ii} \rangle = f_{ii} \langle u_i^2 \rangle, \quad \langle \theta_{\rm p}^2 \rangle = f_\theta \langle \theta_{\rm f}^2 \rangle,$$

$$f_{ii} = 1 - \exp\left(-\frac{T_{ii}^{\rm p}}{\tau_u} \right) = 1 - \exp\left[-\left(\frac{T_{ii}^{\rm p}}{T_{\rm E}} \right) \frac{1}{\Omega_{\rm E}} \right], \quad (39)$$

$$f_\theta = 1 - \exp\left(-\frac{T_\theta^{\rm p}}{\tau_\theta} \right) = 1 - \exp\left[-\left(\frac{T_\theta^{\rm p}}{T_\theta} \right)\left(\frac{T_\theta}{T_{\rm E}} \right) \frac{1}{\Omega_{\rm E}\zeta} \right], \quad (40)$$

$$\Omega_{\rm E} = \tau_u / T_{\rm E}, \quad \zeta = \tau_\theta / \tau_u,$$

where $\Omega_{\rm E}$ is a non-dimensional parameter representing the particle inertia.

The following relation between the functions describing the degree of entrainment of particles into the turbulent velocity and temperature fluctuations of the carrier fluid follow from (39) and (40):

$$f_\theta = 1 - (1 - f_{ii})^{T_\theta^{\rm p}/(\zeta T_{ii}^{\rm p})}. \quad (41)$$

If the integral scales of velocity and temperature fluctuations of the fluid along trajectory of the particle are equal ($T_\theta^{\rm p} = T_{ii}^{\rm p}$), expression (41) is similar to that obtained earlier in [34]. It is seen from (41) that an increase of parameter ζ causes a decrease in the intensity of temperature fluctuations of inertial particles. For inertialess particles $\tau_u, \tau_\theta \to 0$, in the absence of average velocity drift ($W = 0$), an estimation of ratio between Lagrangian and Eulerian time scales of a passive substance versus the integral structural parameters of the flow follows from (36)–(38):

$$\beta_u = T_u^{\rm L}/T_{\rm E} = (1 + \pi\gamma_u^2/2)^{-5/2}, \quad (42)$$

$$\beta_\theta = T_\theta^{\rm L}/T_\theta = (1 + \pi\gamma_\theta^2/2)^{-3/2}, \quad (43)$$

where γ_u, γ_θ are integral structural parameters of turbulent flow

$$\gamma_u = uT_u^{\rm L}/L_{\rm E}, \quad \gamma_\theta = uT_\theta^{\rm L}/L_\theta \quad (44)$$

and $T_u^{\rm L}$, $T_\theta^{\rm L}$ are integral Lagrangian time scales of velocity and temperature fluctuations of the passive substance.

It is obvious, from formulas (42)–(44), that integral Lagrangian time macroscales are always less than the corresponding Eulerian time macroscales determined in the system of coordinates linked with the mean velocity of fluid flow.

The following relations between the integral scales and structural parameters for the Eulerian and Lagrangian correlation functions follows from (42)–(44):

$$\delta_u = uT_{\rm E}/L_E = \gamma_u/\beta_u, \quad \delta_\theta = uT_\theta/L_\theta = \gamma_\theta/\beta_\theta. \quad (45)$$

From Eqs. (16), (17) and (39), (40) we carry out the expressions for integral Lagrangian scales of velocity and temperature fluctuations of particles

$$\Gamma_{ii} = T_{ii}^{\rm p}/[1 - \exp(-T_{ii}^{\rm p}/\tau_u)],$$
$$\Gamma_\theta = T_\theta^{\rm p}/[1 - \exp(-T_\theta^{\rm p}/\tau_\theta)]. \quad (46)$$

This relation shows that Lagrangian time scales of velocity and temperature fluctuations of particles with small inertia $\tau_u \ll T_{\rm E}$, $\tau_\theta \ll T_\theta$ coincide with the Lagrangian time scales of a passive substance. For particles with high inertia ($\tau_u \gg T_{\rm E}, \tau_\theta \gg T_\theta$) the Lagrangian time scales of particles are close to the timescales of their dynamic and thermal relaxation. An increase in the averaged relative velocity of the dispersed phase also leads to the conclusion that the Lagrangian time scales of particles tend to their relaxation times.

Estimates of integral parameters of the Eulerian correlation functions in (30) and (31) are necessary to perform numerical calculation of this equation. An analysis of the relation between the integral scales of velocity and temperature fluctuations based on the more accurate spectral von Karman representation is given in Appendix D.

5. Turbulent diffusion coefficient of inertial particles

The intensity of diffusion of a substance in a turbulent flow is characterized by the averaged value of the product of actual displacement of particles

$$\langle X_{pi}^2(t) \rangle = \int_0^t {\rm d}s_1 \int_0^t {\rm d}s_2 \langle v_{pi}(s_1) v_{pi}(s_2) \rangle.$$

The definition of the turbulent diffusion coefficient of the substance was given by Taylor [37]

$$D_{ii}^{\rm p} = \frac{1}{2}\frac{\rm d}{{\rm d}t} \langle X_{pi}(t) X_{pi}(t) \rangle = \int_0^t \langle v_{pi}(t) v_{pi}(t+s) \rangle\, {\rm d}s. \quad (47)$$

If inequality $t \gg (T_{\rm E}, \tau_u, \tau_\theta)$ is satisfied, then the energy of random motion and the coefficient of turbulent diffusion of particles achieve the stationary values. The coefficient of turbulent diffusion of the dispersed phase can be expressed in terms of energy of random motion of particles and turbulent energy of the carrier phase with the help of Eqs. (3), (12), (14)

$$D_{ii}^{\rm p} = \Gamma_{ii} \langle v_{pi}^2 \rangle = T_{ii}^{\rm p} \langle u_i^2 \rangle. \quad (48)$$

At $\tau_u \to 0$ Eq. (48) yields the expression for the coefficient of turbulent diffusion of a passive substance

$$D_{ii}^{\circ} = T_u^{\rm L} \langle u_i^2 \rangle. \quad (49)$$

The following formula for the ratio between coefficients of turbulent diffusion of the dispersed phase and the passive substance is obtained from (36), (37), (48), and (49)

$$D^{\rm p}_{ii}/D^{\circ}_{ii} = \beta_u^{-1} T^{\rm p}_{ii}/T_{\rm E}. \tag{50}$$

For inertial particles $\tau_u \gg T_{\rm E}$ in Eqs. (34) and (35) parameters $\langle \Lambda^2_{ii}\rangle \to 0$, $\eta_{ii} \to 1$, and in absence of any velocity drift between phases $T^{\rm p}_{ii} \to T_{\rm E}$. In this case the coefficient of turbulent diffusion of the dispersed phase reaches a maximum value, which exceeds the coefficient of turbulent diffusion of the passive substance

$$D^{\rm p}_{ii}/D^{\circ}_{ii}|_{\max} = \beta_u^{-1} > 1.$$

It is seen from (36), (37) and (50) that an increase in the averaged relative velocity between phases leads to the decrease in the time scale of velocity fluctuation functions of carrier phase along the particle trajectory. Hence, it causes a decrease of the coefficient of turbulent diffusivity of dispersed phase (effect of "crossing trajectories").

If the integral Lagrangian and Eulerian time scales are equal, $\beta_u = 1$ (for $\eta_i = 1$ and $W = 0$ we have $T^{\rm p}_{ii} = T_{\rm E}$), then the turbulent diffusion coefficients of dispersed and carrier phases are equal. This is in agreement with so-called Chen's theorem (for reference see [13]). In this case the dependence of turbulent diffusion coefficient of dispersed phase on the relative drift between phases coincides, as seen from Eq. (36), with the well-known Csanady's formula (see [4])

$$D^{\rm p}_{ii}/D^{\circ}_{ii} = [1 + (W/u)^2 (uT_{\rm E}/L_{\rm E})^2]^{-1/2} = [1 + (\alpha\delta_u)^2]^{-1/2}.$$

6. Eulerian correlation functions in various system of coordinates

Going from a system of coordinates moving with the mean flow velocity $\boldsymbol{U}_0$ into a "laboratory" (fixed) system of coordinates we used the Galilean transformation

$$\boldsymbol{U}'(\boldsymbol{x}, t) = \boldsymbol{U}(\boldsymbol{x} + \boldsymbol{U}_0 t, t) - \boldsymbol{U}_0.$$

Here primed parameters are in the "laboratory" system of coordinates, and nonprimed parameters are in the moving system of coordinates.

The turbulent space–time Eulerian correlation function of velocity fluctuations of the fluid phase in the "laboratory" system of coordinates has the following form:

$$E'_{ii}(\boldsymbol{x}_1, \boldsymbol{x}_2; t_1, t_2) = \langle u'_i(\boldsymbol{x}_1, t_1) u'_i(\boldsymbol{x}_2, t_2)\rangle = \langle u_i(\boldsymbol{x}_1 + \boldsymbol{U}_0 t_1, t_1) u_i(\boldsymbol{x}_2 + \boldsymbol{U}_0 t_2, t_2)\rangle.$$

This expression is simplified for homogeneous and stationary turbulent flow as follows:

$$\langle u_i(\boldsymbol{x}_1 + \boldsymbol{U}_0 t_1, t_1) u_i(\boldsymbol{x}_2 + \boldsymbol{U}_0 t_2, t_2)\rangle = E_{ii}(\boldsymbol{Y} + \boldsymbol{U}_0\xi; \xi),$$
$$\boldsymbol{Y} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \quad \xi = |t_1 - t_2|. \tag{51}$$

The Fourier transformation of expression (51) gives

$$E'_{ij}(\boldsymbol{Y}, \xi) = E_{ij}(\boldsymbol{Y} + \boldsymbol{U}_0\xi, \xi) = \int \hat{E}_{ij}(\boldsymbol{k}, \xi) \exp[-{\rm i}\boldsymbol{k}\cdot(\boldsymbol{Y} + \boldsymbol{U}_0\xi)]\,{\rm d}\boldsymbol{k}.$$

The one-point ($Y = 0$) Eulerian correlation function of velocity fluctuations in the "laboratory" system of coordinates has the following form:

$$E'_{ii}(0, \xi) = \int \hat{E}_{ii}(\boldsymbol{k}, \xi) \exp(-i\boldsymbol{k}\cdot\boldsymbol{U}_0\xi)\,{\rm d}\boldsymbol{k}. \tag{52}$$

Note, that expression (52) is similar to formula (24) for $\langle \Lambda^2_n\rangle = 0$. Approximating Eulerian correlation functions in the system of coordinates linked with the mean fluid flow by expression (30), the following expression for the ratio between Eulerian time scales in the moving and fixed systems of coordinates from (51) can be obtained:

$$\frac{T'_{\rm E}}{T_{\rm E}} = \left[1 + \left(\frac{U_0 T_{\rm E}}{L_{\rm E}}\right)^2\right]^{-1/2} = \left[1 + \left(\frac{\delta_u}{\chi}\right)^2\right]^{-1/2}, \quad \chi = \frac{u}{U_0}, \tag{53}$$

where χ is the ratio between the amplitude of velocity fluctuations and the mean fluid flow velocity.

Formula (53) is written for velocity fluctuations that are parallel to the mean flow velocity of carrier phase.

Formulas (42)–(45) and (53) can serve as estimation of integral Lagrangian and Eulerian time scales in various systems of coordinates.

The so-called Taylor's hypothesis of "frozen turbulence" was used by Shraiber at al. [31] to estimate the ratio between the Lagrangian and Eulerian time scales in the system of coordinates of the flow

$$T^{\rm L}_u \approx L_{\rm E}/u. \tag{54}$$

For significant averaged velocity of fluid flow, from (53) and (54) we receive the following estimation of the ratio β_u between time scales

$$\frac{T'_{\rm E}}{T_{\rm E}} \approx \frac{L_{\rm E}}{U_0 T_{\rm E}} \ll 1, \quad \beta_u = \frac{1}{\chi}\frac{T'_{\rm E}}{T_{\rm E}}. \tag{55}$$

7. Results of calculation

Some experimental data will be used to estimate the ratio between the Lagrangian and Eulerian time scales β_u for various types of flow. Ratios between Eulerian time scales $T_{\rm E}/T'_{\rm E}$ for several flows versus the degree of turbulence χ were measured in [29]. These experimental data (1)–(5) are presented in Fig. 1(a). Some experimental data for the parameter γ_u in the case of isotropic turbulent flow behind a grid were presented in [24] (the points (6) on Fig. 1(c) is original experimental data). The initial experimental data and the results of a

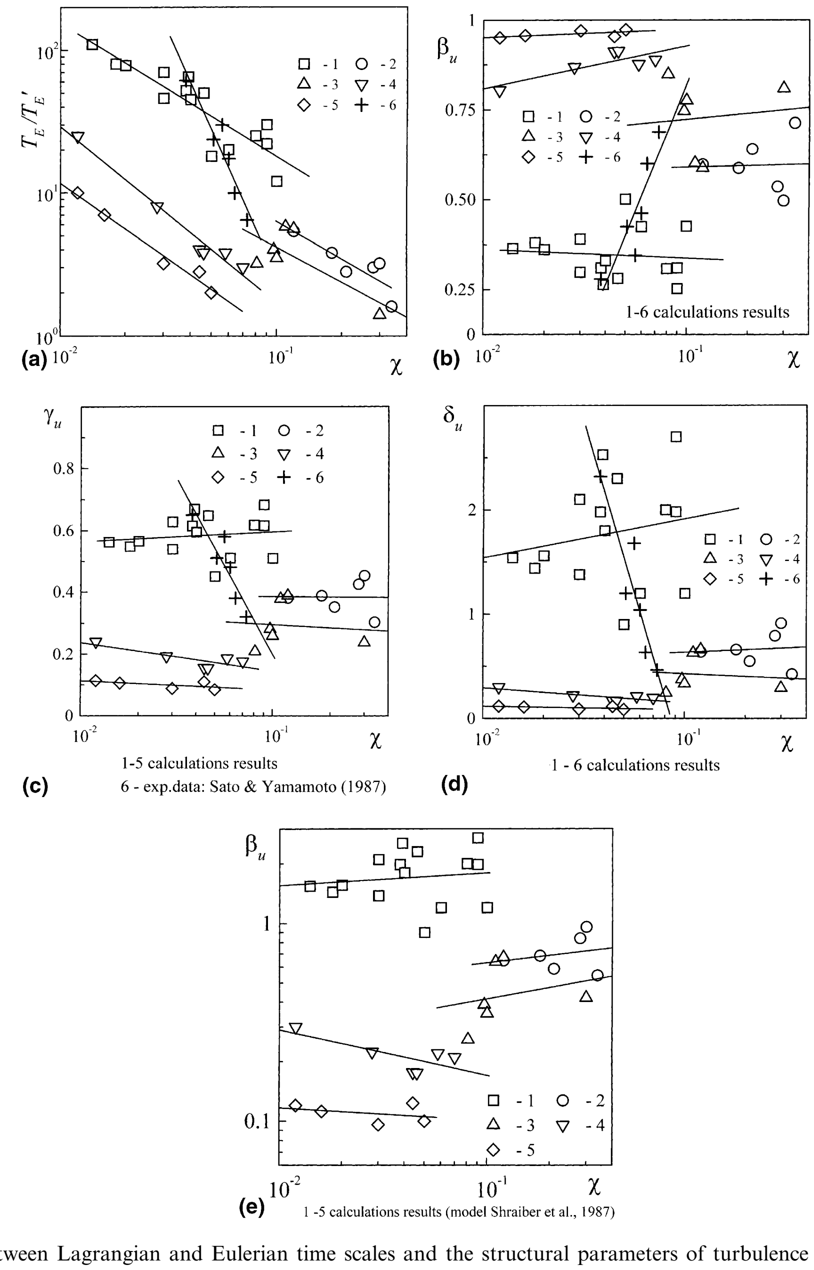

Fig. 1. The ratio between Lagrangian and Eulerian time scales and the structural parameters of turbulence versus the intensity of turbulence for various types of flows in the fixed "laboratory" system and in the system of coordinates connected with the mean velocity of the flow (a)–(d). Calculation of the ratio between the Lagrangian and Eulerian time scales by using the hypothesis of "frozen turbulence" [31] (e). Dots 1–5 on the figure (a) represent the experimental data by Krashenninnikov and Secundov [29]: 1 – turbulent wake behind a cylinder; 2 – mixing zone; 3 – surface atmospheric mixing layer; 4 – tube flow; 5 – the bulk flow in a turbulent jet. Dots 6 in the figure (b) show the results of calculation using experimental data by Sato and Yamamoto [24] for isotropic turbulent flow after a grid. Points 6 in the Fig. 1(c) represent the experimental data by Sato and Yamamoto [24]. In the other figures the dots indicate the results of calculations. The straight lines represent the results of linear interpolation. (a) the ratio between Eulerian time scales in moving and laboratory system of coordinates; (b) the ratio between Lagrangian and Eulerian time scales in the system coordinates moving with mean velocity of the flow; (c) and (d) turbulence structural parameters of velocity fluctuations.

calculation, by Eqs. (42)–(45) and (53), of the ratio between the Lagrangian and Eulerian times β_u, of the structural parameters γ_u, β_u, and δ_u and the ratio T_E/T'_E are presented in Fig. 1. The results of a calculation of the ratio between Lagrangian and Eulerian time scales by the method proposed in [31] are shown in Fig. 1(e). A comparison of Fig. 1(b) and (e) shows a qualitative and quantitative disagreement between results of the calculations of β_u. In [31], the ratio between Lagrangian and Eulerian time scales in the system of coordinates of the flow can be less as well as much larger than unity, and vary in a wide range (see Fig. 1(e)). The results of this paper (Fig. 1(b)) indicate, on the contrary, that the ratio between Lagrangian and Eulerian time scales in the system of coordinates of the flow is always less than unity. In conformity with our calculations $0.3 \leqslant \beta_u < 1$ for all investigated types of flow.

With the exception of the experimental data for γ_u obtained in [24], which change rapidly as the degree of turbulence increases, the parameters γ_u, β_u, and δ_u depend but slightly on the parameter χ and are determined by the type of the flow under study (Fig. 1(c) and (d)). The effect of the type of the flow on the characteristic time scales of turbulence can be estimated using the results of this paper.

A comparison of calculation results with the experimental data [21] on the effect of structural parameters γ_u and δ_u on the ratio between Lagrangian and Eulerian time scales β_u is presented on the Fig. 2. The results of the calculations are compared with the DNS obtained in [30] for the parameters of particles in homogeneous and isotropic turbulence and with the results of experiments performed in [1,2]. The calculation of velocity fluctuations were carried out at $\gamma_u \approx 0.45$, $\beta_u \approx 0.5$. The structural parameters of temperature fluctuations are estimated in accordance to Appendix D. The parameter of dynamic inertia of particles Ω_E, and ratio τ_u/τ_K was found with the help of Eq. (D.13).

The Fig. 3 illustrates a reduction of turbulent diffusion coefficient of particles with an increase of the average velocity drift between phases (parameter α). It is seen that the particles diffusivity in the direction perpendicular to the relative particles velocity is less than that in the longitudinal direction. The experimental data obtained in [1,2] are for the diffusion of particles in the direction perpendicular to relative velocity between phases. It is observed that, as the inertial parameter Ω_E of particle inertia increases, at low drift velocities, the coefficient of turbulent diffusion of the dispersed phase is larger than the diffusion coefficient of the passive substance.

The DNS [30] for two-phase turbulence is conducted neglecting influences of velocity drift on the intensity of random motion of particles. Fig. 4 presents

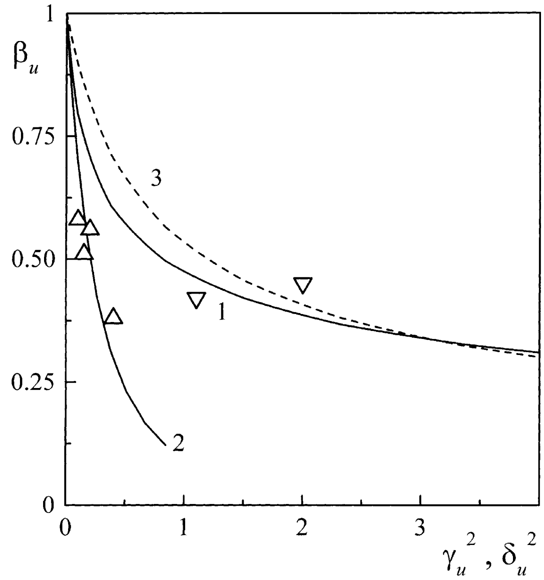

Fig. 2. The ratio between the Lagrangian and Eulerian time scales $\beta_u = T_u^L/T_E$ versus the squared structural parameters. The dots show the experimental data collected in [21], lines are calculation results. 1 – the ratio between the Lagrangian and Eulerian time scales versus the parameter δ_u^2 ($\delta_u = uT_E/L_E$); 2 – the quantity $\gamma_u^2(\gamma_u = uT_u^L/L_E$; 3 – the calculation of the ratio between the time scales β_u versus the parameter δ_u^2 obtained in [21].

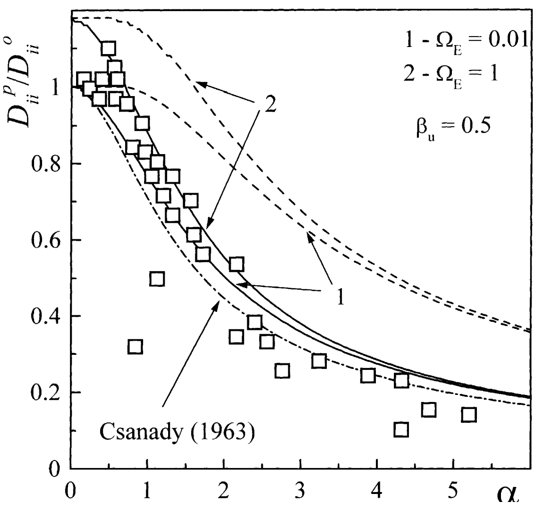

Fig. 3. The effect of the phase relative velocity on the turbulent diffusion coefficient of particles. Dots represent experimental data from [1,2]. Experimental data correspond to the coefficient of dispersed phase diffusion in the direction perpendicular to the velocity drift of phases. The solid lines denote the predictions of diffusion coefficient in the direction perpendicular to the relative velocity, dashed lines show predictions in the direction parallel to the relative velocity. The dotted–dashed line represents the Csanady approximation [4].

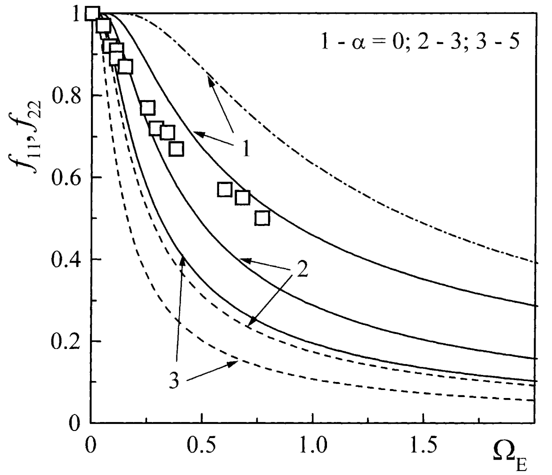

Fig. 4. Functions that represent the degree of entrainment of particles into velocity fluctuations of carrier fluid versus the particles inertia parameter at various drift velocities of phases. Dots show DNS data by Jaberi [30]. DNS data were obtained neglecting the averaged velocity drift of phases ($\alpha = 0$). Lines denote the calculation results for various α. Solid lines are calculation results: solid lines – f_{11}, dashed – f_{22}, dotted–dashed line indicates the results obtained for approximation $T^{\mathrm{p}}_{ii} = T_{\mathrm{E}}$ (formula (36)).

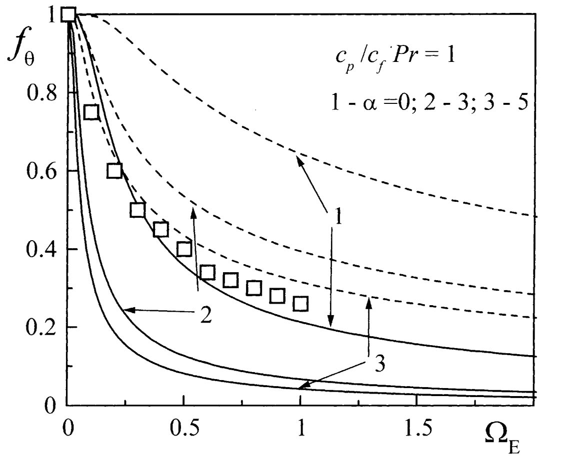

Fig. 5. Function that describes the amplitude of temperature fluctuations of particles versus the inertia parameter at various relative velocities of particles. The dots show the data of DNS by Jaberi [30] at $\alpha = 0$ and $c_{\mathrm{p}}/c_{\mathrm{f}}Pr = 1$. The solid lines denote calculations by formula (37), the dashed lines obtained with the help of formula (37) at $T^{\mathrm{p}}_{\theta} = T_{\mathrm{E}}$.

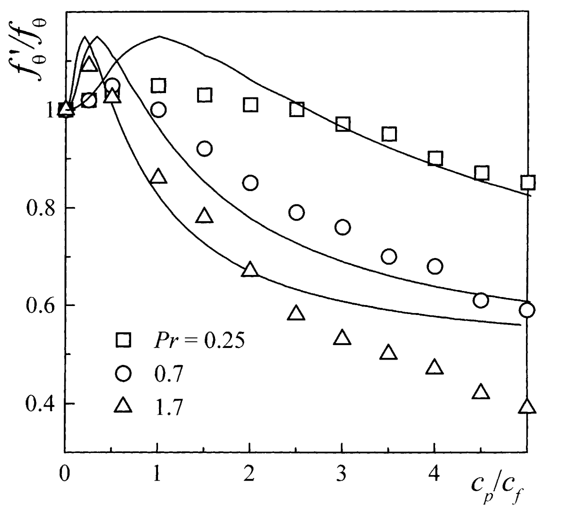

Fig. 6. The ratio between the function f'_{θ} calculated by the formula (38) (model by Yarin and Hetsroni [34]) for $T^{\mathrm{p}}_{\theta} = T^{\mathrm{p}}_{ii}$ and function f_{θ} (37) at various Prandtl numbers ($\Omega_{\mathrm{E}} \approx 0.17$. Dots show the DNS data by Jaberi [30].

the particles response functions (39) with various values of the velocity drift coefficient α. An increase in the drift velocity causes a decrease in the magnitude of fluctuations of particles. The intensity of turbulent motion of particles in the direction parallel to the averaged relative velocity of particles is higher than in the perpendicular direction. It is noticed that the use of the Eulerian time scale alone for evaluation of the amplitude of velocity fluctuations of particles yields results with considerable errors.

It also clear from Fig. 5 that the results of calculations of the function describing the relative temperature fluctuations (40) with the use of the Eulerian time scale alone and taking into account the effect of microstructure are essentially different. As for velocity fluctuations, an increase in the velocity drift leads to a decrease in the magnitude of temperature fluctuations of particles.

Formula (41) represents the squared amplitude of temperature fluctuations of particles in terms of the energy of their random motion. A comparison of the results of a calculation of the relative intensity of temperature fluctuations f'_{θ} calculated at $T^{\mathrm{p}}_{\theta} = T^{\mathrm{p}}_{ii}$ with the DNS data [30] (dots) is shown in Fig. 6. The results of the calculation using the model, proposed in the present paper, are shown by the curves. It is seen that when the multi-scale character of turbulence is taken into account, the parameters of the particles turbulence are predicted with a better accuracy.

8. Conclusions

An analysis of the effects of the microstructure of a turbulent fluid flow on the intensity of velocity and temperature fluctuations of particles taking into account the effect of an average velocity drift between phases, and the relation between thermophysical properties of particles and carrier phase materials was made. For this purpose, a generalization of the so-called hypothesis of "independent averaging" proposed by Corrsin in [28] was used.

It has been shown that the velocity and temperature correlation functions of the carrier fluid along the trajectory of an inertial particle depend on the Eulerian space-time correlation functions measured in the system of coordinates of the fluid flow. For inertialess particles with zero relative velocity (the case of a passive substance), the correlation functions along the micro-particle trajectory coincide with the common Lagrangian correlation functions of turbulence.

An estimation of some integral parameters of the turbulent flow structure based on experimental data was presented. The estimates are related to the type of the flow under study. They make it possible to take into account the microstructure of turbulence in calculations of velocity and temperature fluctuations of particles.

Relations between the time and space integral parameters of Eulerian correlation functions of velocity and temperature for homogeneous isotropic flow have been obtained by using von Karman approximations of the velocity and temperature fluctuation spectra of the carrier fluid.

The effects of dynamic and thermal inertia and the drift velocity of particles on the intensity of random motion and heat transfer in the dispersed phase were investigated. A comparison of the calculation results with the results of DNS and experiments was presented.

Acknowledgements

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Appendix A

A detailed derivation of the expression for probability density function of transition of a particle in a turbulent flow (Eq. (18)) is presented below.

One can define the probability density function of transition of a particle in the space over coordinates, velocities, and temperatures as

$$G_p(\boldsymbol{x},\boldsymbol{V},\Theta,t|\boldsymbol{x}_0,\boldsymbol{V}_0,\Theta_0,t_0) = \delta(\boldsymbol{x}-\boldsymbol{R}_p(t))\delta(\boldsymbol{V}-\boldsymbol{V}_p(t))\delta(\Theta-\Theta_p(t))\delta(\boldsymbol{x}_0 - \boldsymbol{R}_p(t_0))\delta(\boldsymbol{V}_0-\boldsymbol{V}_p(t_0))\delta(\Theta_0-\Theta_p(t_0)). \tag{A.1}$$

Function (A.1) describes the change in the position $\boldsymbol{R}_p(t)$, velocity $\boldsymbol{V}_p(t)$ and temperature $\Theta_p(t)$ of a particle during the time interval $t-t_0$. The initial values of these quantities at t_0 are $\boldsymbol{R}_p(t_0)$, $\boldsymbol{V}_p(t_0)$, $\Theta_p(t_0)$, respectively.

The equation for function (A.1) may be obtained with the help of Eqs. (1) and (2)

$$\frac{\partial G_p}{\partial t} + V_i\frac{\partial G_p}{\partial x_i} + \frac{\partial}{\partial V_i}\left[G_p\frac{U_i(\boldsymbol{x},t)+W_i-V_i}{\tau_u}\right] + \frac{\partial}{\partial \Theta}\left[G_p\frac{\Theta_f(\boldsymbol{x},t)-\Theta_p}{\tau_\theta}\right] = 0. \tag{A.2}$$

The initial condition for Eq. (A.2) has the following form:

$$G_p(\boldsymbol{x},\boldsymbol{V},\Theta,t_0|\boldsymbol{x}_0,\boldsymbol{V}_0,\Theta_0,t_0) = \delta(\boldsymbol{x}-\boldsymbol{x}_0)\delta(\boldsymbol{V}-\boldsymbol{V}_0)\delta(\Theta-\Theta_0). \tag{A.3}$$

One can derive the closed solution of Eq. (A.2) with the initial condition (A.3) in the following form:

$$\begin{aligned}G_p(\boldsymbol{x},\boldsymbol{V},\Theta,t|\boldsymbol{x}_0,\boldsymbol{V}_0,\Theta_0,t_0) &= \exp\left(3\frac{t-t_0}{\tau_u}\right)\exp\left(\frac{t-t_0}{\tau_\theta}\right)\delta\Big\{\boldsymbol{x}-\boldsymbol{x}_0 \\ &\quad + \tau_u(\boldsymbol{V}-\boldsymbol{V}_0) - (t-t_0)\boldsymbol{W} - \int_{t_0}^{t}\boldsymbol{U}(\boldsymbol{R}_p(s),s)\,\mathrm{d}s\Big\} \\ &\quad \times\delta\Big\{\boldsymbol{V}\exp\left(\frac{t-t_0}{\tau_u}\right) - \boldsymbol{V}_0 + \boldsymbol{W}\left(1-\exp\left(\frac{t-t_0}{\tau_u}\right)\right) \\ &\quad - \frac{1}{\tau_u}\int_{t_0}^{t}\exp\left(-\frac{t_0-s}{\tau_u}\right)\boldsymbol{U}(\boldsymbol{R}_p(s),s)\,\mathrm{d}s\Big\} \\ &\quad \times\Big\{\Theta\exp\left(\frac{t-t_0}{\tau_\theta}\right) - \Theta_0 - \frac{1}{\tau_\theta}\int_{t_0}^{t}\exp\left(-\frac{t_0-s}{\tau_\theta}\right) \\ &\quad \times\Theta(\boldsymbol{R}_p(s),s)\,\mathrm{d}s\Big\}.\end{aligned} \tag{A.4}$$

The expression for reduced probability transition function of transition of the particle describes the displacement of the particle at the distance $\boldsymbol{x}-\boldsymbol{x}_0$ during time interval $t-t_0$, and can be obtained from (A.4) in the form

$$\begin{aligned}G_p(\boldsymbol{x},t|\boldsymbol{x}_0,\boldsymbol{V}_0,t_0) &= \int \mathrm{d}\Theta\int \mathrm{d}\boldsymbol{V}G_p(\boldsymbol{x},\boldsymbol{V},\Theta,t|\boldsymbol{x}_0,\boldsymbol{V}_0,\Theta_0,t_0) \\ &= \delta\Big\{\boldsymbol{x}-\boldsymbol{x}_0-\tau_u\boldsymbol{V}_0\left(1-\exp\left(-\frac{t-t_0}{\tau_u}\right)\right) \\ &\quad - \left[(t-t_0)-\tau_u\left(1-\exp\left(-\frac{t-t_0}{\tau_u}\right)\right)\right]\boldsymbol{W} \\ &\quad - \int_{t_0}^{t}\left[1-\exp\left(-\frac{t-t_0}{\tau_u}\right)\right]\boldsymbol{U}(\boldsymbol{R}_p(s),s)\,\mathrm{d}s\Big\}.\end{aligned} \tag{A.5}$$

In the system of coordinates driven with mean velocity of fluid phase $\boldsymbol{U}_0$ we write down

$$\boldsymbol{U}(\boldsymbol{x},t) = \boldsymbol{U}_0 + \boldsymbol{u}(\boldsymbol{x},t), \quad \boldsymbol{V}_0 = \boldsymbol{U}_0 + \boldsymbol{W} + \boldsymbol{v}_0. \tag{A.6}$$

The above expression (A.5) with acceptance of (A.6) yields Eq. (18).

Appendix B

A detailed derivation of the formula for gas velocity fluctuations along the trajectory of a particle is presented. We obtain from Eqs. (18), (19), (21), (22)

$$\begin{aligned}\langle u_i^2\rangle \Psi_{ii}^P(\xi) = \int \mathrm{d}\boldsymbol{v}_0 \int \mathrm{d}\boldsymbol{k}\varphi(\boldsymbol{v}_0)E_{ii}(\boldsymbol{k},\xi)\Big\langle \exp\Big\{ -\mathrm{i}\tau_u \\ \times \left[1-\exp\left(\frac{\xi}{\tau_u}\right)\right]\boldsymbol{k}\cdot\boldsymbol{v}_0 - \mathrm{i}\xi\boldsymbol{W}\cdot\boldsymbol{k} - \mathrm{i}\int_0^\xi \mathrm{d}s \\ \times \left[1-\exp\left(-\frac{\xi-s}{\tau_u}\right)\right]\boldsymbol{k}\cdot\boldsymbol{u}(\boldsymbol{R}_\mathrm{p}(s),s)\Big\}\Big\rangle.\end{aligned} \tag{B.1}$$

After averaging expression (B.1) over an ensemble of turbulent realizations we approximate the velocity fluctuation of the carrier fluid along the particle trajectory $\boldsymbol{u}(\boldsymbol{R}_\mathrm{p}(s),s) = \boldsymbol{u}(s)$ by a random Gaussian process. Its characteristic function is

$$\begin{aligned}&\left\langle \exp\left\{\mathrm{i}\int_0^\xi g_n(s)u_n(s)\,\mathrm{d}s\right\}\right\rangle \\ &\quad = \exp\left\{-\frac{1}{2}\langle u_n^2\rangle \int_0^\xi \mathrm{d}s_1 \int_0^\xi \mathrm{d}s_2 \Psi_{nn}^\mathrm{p} \right. \\ &\qquad \left. \times (|s_1-s_2|)g_n(s_1)g_n(s_2)\right\},\end{aligned} \tag{B.2}$$

where over index n should be summation. The function $g_n(s)$ is chosen in the form

$$g_n(s) = \left[1-\exp\left(-\frac{\xi-s}{\tau}\right)\right]k_n.$$

After integration of the exponential term in the right part of (B.2) we can write the result

$$\begin{aligned}&\left\langle \exp\left\{\mathrm{i}\int_0^\xi g_n(s)u_n(s)\,\mathrm{d}s\right\}\right\rangle \\ &\quad = \exp\left(-\frac{1}{2}k_n^2\langle Y_n^2(\xi)\rangle\right),\end{aligned} \tag{B.3}$$

where the expression for $\langle Y_n^2(\xi)\rangle$ coincides with (27).

Integration of (B.1) over the space of velocity fluctuations of particle with initial velocity distribution of particle (21) leads to formula (26). The correlation functions of fluid phase temperature along the trajectory of a particle in (25) are calculated in a similar way.

Appendix C

The time of dynamic relaxation of particles of spherical shape is defined according to the formula

$$\tau_u = \frac{4}{3}\frac{\rho_\mathrm{p}}{\rho_\mathrm{g}}\frac{d_\mathrm{p}^2}{\nu}\frac{1}{Re_\mathrm{p}C_\mathrm{D}},$$

$$C_\mathrm{D} = \frac{24}{Re_\mathrm{p}}(1+0.179Re_\mathrm{p}^{0.5}+0.013Re_\mathrm{p}).$$

The ratio between temperature and dynamic relaxation times of particles is calculated as follows

$$\frac{\tau_\theta}{\tau_u} = \zeta = \frac{C_\mathrm{D}Re_\mathrm{p}}{24}\frac{c_\mathrm{p}}{c_\mathrm{f}}\frac{3Pr}{Nu}. \tag{C.1}$$

The particle Nusselt number is a function of Prandtl number and the particle Reynolds number (Ranz–Marshall correlation)

$$Nu = 2 + 0.6Re_\mathrm{p}^{0.5}Pr^{0.33}. \tag{C.2}$$

In the calculation of the particle Reynolds number we take into account the sedimentation velocity of the particle and amplitude of relative velocity fluctuations between the particle and the carrier phase

$$Re_\mathrm{p} = Re_\mathrm{p}^\circ[(1-f)^{1/2}+\alpha], \quad Re_\mathrm{p}^\circ = ud_\mathrm{p}/\nu,$$

$$\alpha = W/u, \quad f = \sqrt{\sum_{i=1}^{3} f_{ii}^2/3}, \quad u = \sqrt{\sum_{i=1}^{3} u_i^2/3}.$$

For the Stokes regime of flow around the particles $Re_\mathrm{p} \ll 1$ we have with expressions (C.1) and (C.2) the following relation:

$$\zeta = (3/2)Pr(c_\mathrm{p}/c_\mathrm{f}). \tag{C.3}$$

It is seen from expression (C.3) that an increase in the ratio between the heat capacities of the material of the particles and carrier phase, as well an increase in the Prandtl number of the fluid, leads to an increase in the thermal relaxation time of particles in comparison with the time of dynamic relaxation.

Appendix D

Let us estimate the turbulent integral macroscales of velocity and temperature fluctuations of fluid phase on the basis of a spectral representation of space-time Eulerian correlation functions in the von Karman approximation

$$\hat{E}_u(k,\xi) = \frac{E_u}{k_{\rm E}} g \frac{x^4}{(1+x^2)^{17/6}} \exp\left(-\frac{\xi^2}{T_u^2(k)}\right), \quad x = \frac{k}{k_{\rm E}}, \tag{D.1}$$

$$\hat{E}_\theta(k,\xi) = \frac{E_\theta}{k_\theta} q \frac{y^2}{(1+y^2)^{11/6}} \exp\left(-\frac{\xi^2}{T_u^2(k)}\right), \quad y = \frac{k}{k_\theta}, \tag{D.2}$$

where

$$E_u = \langle u_i u_i \rangle / 2, \quad E_\theta = \langle \theta_{\rm f}^2 \rangle.$$

The turbulent spectra (D.1) and (D.2) have the following normalization:

$$E_\theta = \int \hat{E}_\theta(k,0)\,{\rm d}k, \quad E_u = \int \hat{E}_u(k,0)\,{\rm d}k. \tag{D.3}$$

The characteristic time scales of turbulent eddies, the size of which is of order $\approx k^{-1}$, are terminated by cascade transfer of energy along the spectrums, and look like [32,33]:

$$T_u(k) = \varepsilon_u^{-1/3} k^{-2/3}.$$

From normalization restrictions (D.3) we find the values of constants g, q in the distributions (D.1) and (D.2)

$$g = 2/B(5/2, 1/3), \quad q = 2/B(3/2, 1/3),$$

where

$$B(x,y) = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}}\,{\rm d}t$$

is the Beta function.

From assumed universal behavior of the spectra of velocity and temperature fluctuations in the inertial range $x \gg 1$, $\hat{E}_u(k,0) = C_{\rm K}\varepsilon_u^{2/3}k^{-5/3}$, and in the inertial-connective range $y \gg 1$, $\hat{E}_\theta(k) = C_{\rm B}\varepsilon_\theta\varepsilon_u^{-1/3}k^{-5/3}$, we receive expressions for the scales $k_{\rm E}, k_\theta$ in (D.1) and (D.2)

$$k_{\rm E} = \left(\frac{C_{\rm K}}{g}\right)^{2/3} \frac{\varepsilon_u}{E_u^{3/2}},$$
$$k_\theta = \left(\frac{C_{\rm B}}{q}\right)^{2/3} \frac{(\varepsilon_\theta\varepsilon_u^{-1/3})^{3/2}}{E_\theta^{3/2}}, \tag{D.4}$$

where $C_{\rm K}, C_{\rm B}$ are Kolmogorov and Batchelor constants.

The following expressions for one-dimensional space macroscales of velocity and temperature fluctuations of the continuous phase are found as a result of integration of spectra (D.1) and (D.2):

$$L_{\rm E} = \frac{3\pi}{4E_u} \int k^{-1}\hat{E}_u(k,0)\,{\rm d}k = \frac{3\pi}{8}\frac{g}{k_{\rm E}} B(2, 5/6), \tag{D.5}$$

$$L_\theta = \frac{\pi}{2E_\theta} \int k^{-1}\hat{E}_\theta(k,0)\,{\rm d}k = \frac{\pi}{4}\frac{q}{k_\theta} B(1, 5/6). \tag{D.6}$$

From the Eqs. (D.4)–(D.6), we find expressions for ratio between macroscales of temperature and velocity correlation functions

$$\frac{L_\theta}{L_{\rm E}} = \frac{2}{3}\left(\frac{q}{g}\right)^{5/2} \frac{B(1,5/6)}{B(2,5/6)} \left(\frac{C_{\rm K}}{C_{\rm B}} R\right)^{3/2},$$
$$R = \frac{E_\theta}{\varepsilon_\theta}\frac{\varepsilon_u}{E_u}. \tag{D.7}$$

The integral time scales of velocity and temperature fluctuations of carrier phase and the ratio between them are also established using the spectra (D.1) and (D.2)

$$T_{\rm E} = \frac{1}{E_u}\int {\rm d}k \int_0^\infty {\rm d}\xi \hat{E}_u(k,\xi) = b_{\rm E}\frac{E_u}{\varepsilon_u},$$

$$b_{\rm E} = \frac{\sqrt{\pi}}{C_{\rm K}} \frac{B(13/6, 2/3)}{B^2(5/2, 1/3)}, \tag{D.8}$$

$$T_\theta = \frac{1}{E_\theta}\int {\rm d}k \int_0^\infty {\rm d}\xi \hat{E}_\theta(k,\xi) = b_\theta\frac{E_\theta}{\varepsilon_\theta},$$

$$b_\theta = \frac{\sqrt{\pi}}{C_{\rm B}} \frac{B(7/6, 2/3)}{B^2(3/2, 1/3)}, \tag{D.9}$$

$$T_\theta/T_{\rm E} = R b_\theta / b_{\rm E}. \tag{D.10}$$

The parameters that characterized the internal microstructure of velocity and temperature fluctuations in the Eulerian variables, $\delta_{\rm E} = T_{\rm E}u/L_{\rm E}$, $\delta_\theta = T_\theta u/L_\theta$ are estimated with the help of Eqs. (D.5), (D.6), (D.8), and (D.9).

The Kolmogorov constant is chosen as $C_{\rm K} = 1.65$. In the literature, the values of Batchelor constant vary in the range $C_{\rm B} = 0.85\ldots1.16$. In the present paper we accept the value $C_{\rm B} = 0.85$. The value of parameter R also changes in the range $R = 0.3\ldots1.2$. We elected the estimation $R \approx 0.5$, which is in agreement with experimental data in [35,36]. For these established constants we found

$$L_\theta/L_{\rm E} \approx 0.47, \quad T_\theta/T_{\rm E} \approx 0.78, \quad b_{\rm E} \approx 0.21,$$
$$b_\theta \approx 0.33 \quad \delta_{\rm E} \approx 0.52, \quad \delta_\theta/\delta_{\rm E} \approx 1.65. \tag{D.11}$$

The Reynolds number of turbulence $Re_\lambda = u\lambda/\nu$ is connected with the kinetic energy of turbulent fluctuations as follows (see, as an example [32]):

$$Re_\lambda = \left(\frac{20}{3}\right)^{1/2} \frac{E_u}{(\nu\varepsilon_u)^{1/2}}. \tag{D.12}$$

The ratio between integral time macroscale and Kolmogorov microscale $\tau_K = (\nu/\varepsilon_u)^{1/2}$ follows from (D.8) and (D.12)

$$\frac{T_E}{\tau_K} = \beta_E \left(\frac{3}{20}\right)^{1/2} Re_\lambda. \qquad \text{(D.13)}$$

Eq. (D.13) gives the parameter of particles inertia Ω_E in the terms of particles dynamic relaxation time and Kolmogorov time microscale

$$\Omega_E = \frac{\tau_u}{\tau_K}\left(\frac{20}{3}\right)^{1/2} \frac{1}{\beta_E Re_\lambda}. \qquad \text{(D.14)}$$

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